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Inflation and Unemployment in the Long Run

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Abstract

We study the long-run relation between money, measured by inflation or interest rates, and unemployment. We first document in the data a positive relation between these variables at low frequencies. We then develop a framework where unemployment and money are both modeled using microfoundations based on search and bargaining theory, providing a unified theory for analyzing labor and goods markets. The calibrated model shows that money can account for a sizable fraction of trends in unemployment. We argue it matters, qualitatively and quantitatively, whether one uses monetary theory based on search and bargaining, or an alternative ad hoc specification.

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1 Introduction

We study the relationship between monetary policy, as measured by inflation or nominal interest rates, and labor market performance, as measured by unemployment. While this is an old issue, our focus differs from the existing literature by concentrating on the longer run – we are less interested in business cycles, and more in relatively slowly moving trends.¹ One reason to focus on the longer run is that it may well be more important from a welfare and policy perspective. Many macroeconomists seem obsessed with increases in unemployment, say, over the business cycle; we want to redirect attention to what happens at lower frequencies, since avoiding a bad decade, like the 1970s, from a labor market perspective, probably matters a lot more than smoothing out any given recession.

Another reason to focus on the long run is that economic theory has much cleaner implications for what happens at lower frequencies, which are less likely to be clouded by complications such as signal extraction problems and other forms of imperfect information, or nominal stickiness and other rigidities. We abstract from such complications to focus on the effect of inflation on the cost of carrying real balances for transactions purposes. As Friedman (1977) put it: “There is a natural rate of unemployment at any time determined by real factors. This natural rate will tend to be attained when expectations are on average realized. The same real situation is consistent with any absolute level of prices or of price change, *provided allowance is made for the effect of price change on the real cost of holding money balances*” (emphasis added). This is the effect studied here.

To begin, we want to know the facts about the relation between nominal variables and the labor market. Using quarterly U.S. data from 1955-2005, Fig. 1.1 shows

¹The standard way to define business cycle phenomena in modern macro (see e.g. the Cooley 1995 volume) is this: take a given time series y_t ; apply the HP (or some other) filter to get the trend y_t^T ; and define the cyclical component by the deviation $y_t^D = y_t - y_t^T$. Rather than y_t^D , the object of interest in this study is y_t^T . This is not to say our model does not make predictions about high-frequency behavior – an equilibrium generates y_t for all t – but we are more confident about the predictions for y_t^T because we abstract from some effects that may be relevant at higher frequencies, as discussed below.

scatter plots between inflation and unemployment, progressively removing more of the higher frequency as we move through the panels by applying stronger HP filters. The last panel alternatively filters the data using five-year averages. It is clear that after filtering out the higher frequencies, there is a strong positive relationship between the relatively slowly moving trends in these variables. Fig. 1.2 shows a similar pattern using nominal (Aaa corporate bond) interest rates instead of inflation.² Fig. 1.3 shows the time series instead of scatter plots. We conclude that (i) movements in trend unemployment are large, and (ii) they are positively correlated with the trends in the nominal interest and inflation rates. This is true for the period as a whole, even if the relation sometimes goes the other way in the shorter run, including the 60s where a downward sloping Phillips curve is evident.³

We want to know how much we can account for in these observations using basic economic theory. To this end, we build a general equilibrium model of unemployment and money demand based on search frictions in labor and goods markets, abstracting from nominal misperceptions and rigidities. As suggested by Friedman, to understand the impact of monetary policy on the natural rate of unemployment, it seems important to incorporate the effect of inflation on the cost of holding real balances, which means we need a theory where the cost of holding money and hence the benefit of holding money are made explicit. Additionally, it would seem good to have a theory of unemployment that has proven successful in other contexts.

²This is no surprise, given the Fisher equation, which says nominal rates move one-for-one with inflation, *ceteris paribus*. In the working paper Berentsen et al. (2008), and on the web at <http://www.wwz.unibas.ch/witheo/aleks/BMWII/BMWII.html>, we update Lucas (1980) to show the Fisher equation and quantity equations continue to hold up in the long run with more recent data. The quantity equation suggests we should get similar pictures using money growth instead of inflation or interest rates, and we show this is true, using $M0$, $M1$ or $M2$. We also make the same point using different interest rates, including the T-Bill rate, using employment rather than unemployment, and using an extended sample.

³See Beyer and Farmer (2007), Huag and King (2008) and the references therein for more formal analyses of the data than we can present here. Huag and King in particular apply a band-pass filter (as discussed in Christiano and Fitzgerald 2003) to the same data, and find a positive relationship between unemployment and inflation for bands longer than the typical business cycle. They also tested for multiple structural change at unknown dates. They conclude, “After accounting for breaks, the sub-periods lead us to the same conclusion that the long run association of unemployment with inflation is positive. Although we used different and more formal methods, our findings support the position in BMW.”

In recent years, much progress has been made studying both labor and monetary economics using theories that explicitly incorporate frictions – in particular, search and matching frictions, noncompetitive pricing, anonymity and imperfect monitoring, etc. Models with frictions are natural for understanding dynamic labor markets and hence unemployment, as well as goods markets and the role of money. However, existing papers analyze either unemployment *or* money in isolation. One objective here is to provide a framework that allows us to analyze unemployment *and* money in an environment with logically consistent microfoundations. Although there are various ways to proceed, in terms of different approaches in the literature, here we integrate Mortensen and Pissarides (1994) with Lagos and Wright (2005). The result is a tractable model that makes sharp predictions about several interesting effects, including the positive impact of inflation or interest rates on unemployment.⁴

We then consider the issue quantitatively by calibrating the model and asking how it accounts for the above-mentioned observations. Suppose for the sake of a controlled experiment that monetary policy is the only driving force over the period – i.e. assume counterfactually that demographics, productivity, fiscal policy, etc. were constant. Given monetary policy behaved as it did, how well can we account for movements in trend unemployment? We find that the model accounts for a sizable fraction of the lower-frequency movement in unemployment as a result of observed changes in trend inflation and interest rates. For instance, monetary policy alone can generate around half of the 3 point increase in trend unemployment in the 70s, and about the same fraction of the decline in the 80s. Money matters. However, we also ask how this prediction is affected by innovations in payments, and conclude that in the future money may matter less for unemployment.

Finally, we argue that it makes a difference that we model money using search-and-bargaining theory, as opposed to an ad hoc cash-in-advance constraint, as follows.

⁴In his study of the Fisher and quantity equations, Lucas (1980) warns against making much of a pattern between filtered inflation and unemployment, given the argument in Friedman (1968) and Phelps (1969) that the long-run Phillips curve must be vertical. Following Friedman (1977), our position is that a positive relation between inflation and unemployment is as much “an implication of a coherent economic theory” as Lucas said the Fisher and quantity equations are.

First, we consider a version where the goods market is frictionless except for cash-in-advance, and show analytically that the channels through which money matters are different than in our model. Second, not only are the channels qualitatively different, we show that search-and-bargaining frictions are key to quantitatively accounting for the observations of interest. Hence, while we like our framework because labor and commodity markets are both modeled using logically consistent principles, this is not just a matter of aesthetics – the substantive predictions of a model with these detailed microfoundations are different from what one gets using an ad hoc approach.

The rest of the paper is organized as follows. Sections 2 and 3 describe the model and solve for equilibrium. Section 4 presents the quantitative analysis. Section 5 compares our model with cash-in-advance, and Section 6 concludes.⁵

2 The Basic Model

Time is discrete and continues forever. Each period, there are three distinct markets where economic activity takes place: a labor market in the spirit of Mortensen-Pissarides; a goods market in the spirit of Kiyotaki-Wright; and a general market in the spirit of Arrow-Debreu. We call these the MP, KW and AD markets, and assume MP convenes first, then KW, then AD. As shown in Lagos-Wright, alternating KW and AD markets makes the analysis much more tractable than, say, a model with only KW markets, and we take advantage of that here. There are two types of agents, firms and households, indexed by f and h . The set of h is $[0, 1]$; the set of f is arbitrarily large, but not all are active at any point in time. Households work, consume, and enjoy utility; firms maximize profits and pay dividends.

⁵Other recent attempts to bring monetary issues to bear on search-based labor models include Farmer (2005), Blanchard and Gali (2005), and Gertler and Trigari (2006), but they impose nominal rigidities, which we think are less relevant for longer-run issues. Lehmann (2006), Shi (1998,1999) and Shi and Wang (2006) are closer to our approach, although the details are different. Rocheteau et al. (2006) and Dong (2007) use similar monetary economics, but a different theory of unemployment, Rogerson’s (1988) indivisible labor model; while that leads to some interesting results, there are reasons to prefer Mortensen-Pissarides. Earlier, Cooley and Hansen (1989) stuck a cash-in-advance constraint into Rogerson, as Andofatto et al. (2003) and Cooley and Quadrini (2004) do to Mortensen-Pissarides. As mentioned, we discuss cash-in-advance models in Section 5.

As in any MP-type model, h and f can match bilaterally to create a job, and e indexes employment status: $e = 1$ if an agent is matched and $e = 0$ otherwise. We define a value function for each market, $U_e^j(z)$, $V_e^j(z)$ and $W_e^j(z)$, which depend on type $j \in \{h, f\}$, status $e \in \{0, 1\}$, real balances $z \in [0, \infty)$, and, in general, aggregate state variables, but in the benchmark model fundamentals are constant and we focus on steady states, so aggregate states are subsumed in the notation.⁶ We adopt the following convention for measuring real balances. When an agent brings in m dollars to the AD market, we let $z = m/p$, where p is the current price level. He then takes $\hat{z} = \hat{m}/p$ out of that market and into the next period. In the next AD market the price level is \hat{p} , so the real value of the money is $\hat{z}\hat{p}$, where $\hat{p} = p/\hat{p}$ converts \hat{z} into units of the numeraire good x in that market.

2.1 Households

A household h in the AD market solves

$$\begin{aligned} W_e^h(z) &= \max_{x, \hat{z}} \{x + (1 - e)\ell + \beta U_e^h(\hat{z})\} \\ \text{s.t. } x &= ew + (1 - e)b + \Delta - T + z - \hat{z} \end{aligned} \quad (1)$$

where x is consumption, ℓ the utility of leisure, w the wage, b UI benefits, Δ dividend income, T a lump-sum tax, and β a discount factor (without loss in generality, h discounts between periods but not across markets within a period). Notice w is paid in AD, even though matching occurs in MP. Eliminating x from the budget equation,

$$W_e^h(z) = I_e + z + \max_{\hat{z}} \{-\hat{z} + \beta U_e^h(\hat{z})\}, \quad (2)$$

where $I_e = ew + (1 - e)(b + \ell) + \Delta - T$.

This immediately implies the usual simplification in LW-type models: W_e^h is linear in z and I_e , and the choice of \hat{z} is independent of z and I_e . Although it looks like

⁶For matched agents, the wage w is also a state variable, since it is set in MP and carried forward to KW and AD; to reduce clutter this is also subsumed in the notation. In the Appendix, where policy and productivity follow stochastic processes and unemployment varies endogenously over time, we keep track of these plus w as state variables.

\hat{z} could depend on e through U_e^h , we will see below that $\partial U_e^h / \partial \hat{z}$ and hence \hat{z} are actually independent of e . This means that every h exits the AD market with the same \hat{z} , at least given an interior solution for x , which holds if $b + \ell$ is not too small. These results require quasi-linearity, which is valid here because utility is linear in the numeraire good x , as in any standard MP model.⁷

In KW, another good q is traded, which gives utility $v(q)$, with $v(0) = 0$, $v' > 0$ and $v'' < 0$. In this market, agents trade bilaterally, and we assume at least some meetings are anonymous to generate a role for a medium of exchange. Thus, suppose h asks f for q in KW and promises to pay later, say, in the next AD market. If f does now know who h is, the latter can renege without fear of repercussion; so f insists on quid pro quo. If h cannot store or transport x , money has a role (see Kocherlakota 1997, Wallace 2001, Araujo 2004, and Aliprantis et al. 2007 for formal discussions). To make money essential we only need *some* anonymous meetings – we need not rule out *all* credit. Let ω denote the probability a random match is anonymous. For now, as a benchmark, we set $\omega = 1$ and return to the general case in Section 4.3.⁸

For h in the KW market,

$$V_e^h(z) = \alpha_h v(q) + \alpha_h W_e^h[\rho(z - d)] + (1 - \alpha_h) W_e^h(\rho z), \quad (3)$$

where α_h is the probability of trade and (q, d) the terms of trade, to be determined below. Using the linearity of W_e^h , we can simplify this to

$$V_e^h(z) = \alpha_h [v(q) - \rho d] + W_e^h(0) + \rho z. \quad (4)$$

The probability α_h is given by a CRS matching function \mathcal{M} : $\alpha_h = \mathcal{M}(B, S)/B$, where B and S are the measures of buyers and sellers in KW. Letting $Q = B/S$ be

⁷In fact, we get a degenerate distribution of \hat{z} as long as AD utility is $x + \Upsilon_e(\mathbf{x})$, where \mathbf{x} is a vector of other goods. Also, a recent extension by Liu (2009) allows the employed and unemployed to value KW goods differently, leading to a two-point distribution, without complicating things much.

⁸The case $\omega = 0$ is also of interest, embedding as it does a genuine retail sector, albeit a cashless one, into the standard MP model. That case can be used to study many interesting interactions between commodity and labor markets, including the effects of goods market regulation, sales taxes, etc. on employment. One can also make ω endogenous, as in related models by Dong (2009), where it is a choice of h , and Lester et al. (2009), where it is a choice of f . This is perhaps especially important for large changes in inflation, where one might expect ω not to be invariant; for now ω is fixed, but in Section 4.3 we allow it to change over time.

the queue length, or market tightness, $\alpha_h = \mathcal{M}(Q, 1)/Q$. We assume that $\mathcal{M}(Q, 1)$ is strictly increasing in Q , with $\mathcal{M}(0, 1) = 0$ and $\mathcal{M}(\infty, 1) = 1$, and $\mathcal{M}(Q, 1)/Q$ is strictly decreasing with $\mathcal{M}(0, 1) = 1$ and $\mathcal{M}(\infty, 1) = 0$, as is true for most standard matching functions (see e.g. Menzio 2007).

In equilibrium, every h participates in KW, so $B = 1$, and moreover every h is identical from the viewpoint of f since they all have the same amount of money. However, f can only participate in KW if $e = 1$, since an unmatched firm has nothing to sell (given inventories are liquidated in AD as discussed below). Thus, $\alpha_h = \mathcal{M}(1, 1-u)$, where u is unemployment entering KW. This establishes a first connection between the goods and labor markets: consumers are better off in the former when times are better, in the sense that u is lower, in the latter, both because the probability of a trade is higher, and, in equilibrium, because this affects the terms of trade.

For h in the MP market,

$$U_1^h(z) = V_1^h(z) + \delta [V_0^h(z) - V_1^h(z)] \quad (5)$$

$$U_0^h(z) = V_0^h(z) + \lambda_h [V_1^h(z) - V_0^h(z)], \quad (6)$$

where δ is the job destruction rate and λ_h the job creation rate. Job destruction is exogenous, but job creation is determined by another matching function \mathcal{N} : $\lambda_h = \mathcal{N}(u, v)/u = \mathcal{N}(\tau, 1)/\tau$, where $\tau = v/u$ is labor market tightness, with u unemployment and v vacancies (one has to distinguish between ‘vee’ v for vacancies and ‘upsilon’ v for utility, but it is always clear from the context). We make assumptions on \mathcal{N} similar to \mathcal{M} . Wages are determined when f and h meet in MP, although they are paid in the next AD market, and in on-going matches w can be renegotiated each period.

It is sometimes convenient to summarize the three markets by one equation. Substituting $V_e^h(z)$ from (4) into (5) and using the linearity of W_e^h ,

$$U_1^h(z) = \alpha_h [v(q) - \rho d] + \rho z + \delta W_0^h(0) + (1 - \delta)W_1^h(0)$$

Something similar can be done for U_0^h . Inserting into (2), in steady state, the AD

problem becomes

$$W_e^h(z) = I_e + z + \max_{\hat{z}} \{-\hat{z} + \beta\alpha_h[v(q) - \rho d] + \beta\rho\hat{z}\} + \beta\mathbb{E}W_e^h(0) \quad (7)$$

where the expectation is wrt next period's employment status conditional on e . We claim the KW terms of trade (q, d) may depend on \hat{z} but not on employment status – see Section 3.1. Hence, from (7), the choice \hat{z} is independent of e , as well as I_e and z .⁹

2.2 Firms

Firms carry no money out of AD. In the MP market, we have

$$U_1^f = \delta V_0^f + (1 - \delta)V_1^f \quad (8)$$

$$U_0^f = \lambda_f V_1^f + (1 - \lambda_f)V_0^f, \quad (9)$$

where $\lambda_f = \mathcal{N}(u, v)/v = \mathcal{N}(1, \tau)/\tau$. This is standard. Where we deviate from textbook MP theory is that, rather than having f and h each consume a share of the output, here f takes it to the goods market and looks to trade with another h . Thus, in the model, as in reality, households do not necessarily consume what they make each day at work. Output in a match is denoted y , and measured in units of the AD good. If f sells q units in KW, there is a transformation cost $c(q)$, with $c' > 0$ and $c'' \geq 0$, so that $y - c(q)$ is left over to bring to the next AD market.¹⁰

⁹Recall that KW meetings are anonymous with probability $\omega = 1$ in this benchmark. More generally, the maximand in (7) should be

$$-\hat{z} + \beta\alpha_h\omega[v(q^m) - \rho d^m] + \beta\alpha_h(1 - \omega)[v(q^c) - \rho d^c] + \beta\rho\hat{z}$$

where (q^m, d^m) and (q^c, d^c) are the terms of trade in money and credit meetings, respectively. The crucial difference is that money trades are constrained by $d^m \leq \hat{z}$ while no such constraint applies to credit trades. This implies the choice of \hat{z} is actually independent of (q^c, d^c) . In fact, most of the predictions are exactly the same for all values of $\omega > 0$ as long as we adjust α_h so $\alpha_h\omega$ is constant. See Section 4.3 for more on the model with $\omega < 1$.

¹⁰We also solved the model where output is in KW goods, and there is a technology for transforming unsold KW goods into AD goods. The results are essentially the same. One can alternatively assume unsold KW goods are carried forward to the next KW market, but having f liquidate inventory in AD avoids the problem of tracking inventories across f , just like the AD market allows us to avoid tracking the distribution of money across h .

For f in KW,

$$V_1^f = \alpha_f W_1^f [y - c(q), \rho d] + (1 - \alpha_f) W_1^f(y, 0) \quad (10)$$

where $\alpha_f = \mathcal{M}(B, S)/S$. The AD value of f with inventory x , real balances z , and wage commitment w is $W_1^f(x, z) = x + z - w + \beta U_1^f$. Thus,

$$V_1^f = R - w + \beta \left[\delta V_0^f + (1 - \delta) V_1^f \right], \quad (11)$$

where $R = y + \alpha_f[\rho d - c(q)]$ is expected revenue. Obviously, the KW terms of trade (q, d) affect R , and hence in equilibrium affect entry and employment, establishing another link between goods and labor markets. And as long as f derives at least some revenue from cash transactions, monetary factors affect labor market outcomes.

To model entry, as is standard, any f with $e = 0$ can pay k in units of x in the AD market to enter the next MP market with a vacancy. Thus

$$W_0^f = \max \left\{ 0, -k + \beta \lambda_f V_1^f + \beta(1 - \lambda_f) V_0^f \right\},$$

where $V_0^f = W_0^f = 0$ by free entry. Thus $k = \beta \lambda_f V_1^f$, which by (11) implies

$$k = \frac{\beta \lambda_f (R - w)}{1 - \beta(1 - \delta)}. \quad (12)$$

Profit over all firms is $(1 - u)(R - w) - vk$, which they pay out as dividends. If the representative h holds the representative portfolio (say, shares in a mutual fund) this gives equilibrium dividend income Δ .

2.3 Government Policy

Government consumes G , pays UI benefit b , levies tax T , and prints money at rate π , so that $\hat{M} = (1 + \pi)M$, and π equals inflation in steady state. The budget constraint $G + bu = T + \pi M/p$ holds at every date, without loss of generality, by Ricardian equivalence. For steady state analysis, we can equivalently describe monetary policy in terms of setting the nominal interest rate i or π , by virtue of the Fisher equation $1 + i = (1 + \pi)/\beta$. In the stochastic model in the Appendix we specify policy in terms of interest rate rules. We always assume $i > 0$, although one can take the limit as $i \rightarrow 0$, which is the Friedman rule.

3 Equilibrium

We assume that agents are price takers in the AD market, and bargain over the terms of trade in MP and KW.¹¹ Given this, we determine steady state equilibrium as follows. First, taking unemployment u as given, we solve for the value of money q as in Lagos-Wright (2005). Then, taking q as given, we solve for u as in Mortensen-Pissarides (1996). If we depict these results in (u, q) space as the LW curve and MP curve, their intersection determines equilibrium unemployment and the value of money, from which all other variables easily follow.

3.1 Goods Market Equilibrium

When f and h meet in KW, the terms of trade (q, d) are determined by the generalized Nash bargaining solution

$$\max_{q,d} [v(q) - \rho d]^\theta [\rho d - c(q)]^{1-\theta}, \quad (13)$$

s.t. $d \leq z$ and $c(q) \leq y$, which say the parties cannot leave with negative cash balances or inventories. The first term in (13) is the surplus of h and the second the surplus of f , using the linearity of W_e^j , while θ is the bargaining power of h . We assume $c(q) \leq y$ is not binding. As established in Lagos-Wright, in any equilibrium, the solution of (13) involves $d = z$ and $q = g^{-1}(\rho z)$, where

$$g(q) \equiv \frac{\theta c(q)v'(q) + (1-\theta)v(q)c'(q)}{\theta v'(q) + (1-\theta)c'(q)}. \quad (14)$$

Notice $\partial q / \partial z = \rho / g'(q) > 0$, so bringing more money gets h more KW goods, but nonlinearly (unless $\theta = 1$ and c is linear).

Given the bargaining outcome $d = z$ and $q = g^{-1}(\rho z)$, we can rewrite the choice of \hat{z} by h in AD as

$$\max_{\hat{z} \geq 0} \{-\hat{z} + \beta \alpha_h v[g^{-1}(\rho z)] + \beta(1 - \alpha_h)\rho \hat{z}\}. \quad (15)$$

¹¹In Berentsen et al. (2008) we consider alternative pricing mechanisms for both MP and KW, including price taking and price posting. Here we focus on bargaining because it is easy and standard in the literatures on search unemployment and money.

The solution is given by

$$\frac{1}{\beta\rho} = \alpha_h \frac{v'(q)}{g'(q)} + 1 - \alpha_h. \quad (16)$$

Using $1/\beta\rho = 1 + i$ and $\alpha_h = \mathcal{M}(1, 1 - u)$, we get

$$\frac{i}{\mathcal{M}(1, 1 - u)} = \frac{v'(q)}{g'(\hat{q})} - 1. \quad (17)$$

This is the LW curve, determining q as in Lagos-Wright, except there α_h was fixed and here $\alpha_h = \mathcal{M}(1, 1 - u)$. Its properties follow from well-known results. For instance, simple conditions guarantee the RHS of (17) is monotone, and hence there is a unique solution $q > 0$, with $\partial q/\partial u < 0$.¹² Intuitively, the higher is unemployment the lower is the probability that h matches in KW, which lowers the demand for money, and reduces its value q . Also, given u , (17) implies q is decreasing in i . These and other properties of the LW curve are summarized below.

Proposition 1 *Let q^* solve $v'(q^*) = c'(q^*)$. For all $i > 0$ the LW curve slopes downward in (u, q) space, with $u = 0$ implying $q \in (0, q^*)$ and $u = 1$ implying $q = 0$. The curve shifts down with i and up with θ . As $i \rightarrow 0$, $q \rightarrow q_0$ for all $u < 1$, where q_0 is independent of u , and $q_0 = q^*$ iff $\theta = 1$.*

3.2 Labor Market Equilibrium

In MP, we use Nash bargaining over w with threat points given by continuation values and η the bargaining power of f . It is routine to solve for

$$w = \frac{\eta [1 - \beta (1 - \delta)] (b + \ell) + (1 - \eta) [1 - \beta (1 - \delta - \lambda_h)] R}{1 - \beta (1 - \delta) + (1 - \eta) \beta \lambda_h}, \quad (18)$$

exactly as in Mortensen-Pissarides. Substituting this and $R = y + \alpha_f[\rho d - c(q)]$ into (12), the free entry condition becomes

$$k = \frac{\lambda_f \eta [y - b - \ell + \alpha_f (\rho d - q)]}{r + \delta + (1 - \eta) \lambda_h}. \quad (19)$$

¹²Conditions that make the RHS of (17) monotone include decreasing absolute risk aversion, or $\theta \approx 1$. Alternatively, the analysis in Wright (2009) implies there is generically a unique solution with $\partial q/\partial u < 0$ even if the RHS is not monotone.

To simplify (19), use the steady state condition $(1 - u)\delta = N(u, v)$ to implicitly define $v = v(u)$ and write $\alpha_f = \mathcal{M}(1, 1 - u)/(1 - u)$, $\lambda_f = \mathcal{N}[u, v(u)]/v(u)$ and $\lambda_h = \mathcal{N}[u, v(u)]/u$. Using these plus $\rho d = g(q)$, (19) becomes

$$k = \frac{\eta \frac{\mathcal{N}[u, v(u)]}{v(u)} \left\{ y - b - \ell + \frac{\mathcal{M}(1, 1 - u)}{1 - u} [g(q) - c(q)] \right\}}{r + \delta + (1 - \eta) \frac{\mathcal{N}[u, v(u)]}{u}}. \quad (20)$$

This is the MP curve, determining u as in Mortensen-Pissarides, except the total surplus (the term in braces) includes not just $y - b - \ell$ but also the expected surplus from KW trade. Routine calculations show the MP curve is downward sloping. Intuitively, when q is higher, profit and hence the benefit from opening a vacancy are higher, so ultimately unemployment is lower. Also, given q , u is increasing in b , ℓ and k and decreasing in y . These and other properties of the MP curve are summarized below, under a maintained assumption $k(r + \delta) < \eta[y - b - \ell + g(q^*) - c(q^*)]$, without which the market shuts down.

Proposition 2 *The MP curve slopes downward in (u, q) space and passes through (\underline{u}, q^*) , where $\underline{u} \in (0, 1)$. If $k(r + \delta) \geq \eta(y - b - \ell)$ it passes through $(1, \underline{q})$, where $\underline{q} > 0$, and if $k(r + \delta) < \eta(y - b - \ell)$ it passes through $(\bar{u}, 0)$, where $\bar{u} > 0$. It shifts to the right with b , ℓ and k , and to the left with y .*

3.3 General Equilibrium

The LW and MP curves both slope downward in the rectangle $\mathcal{B} = [0, 1] \times [0, q^*]$ in (u, q) space, as shown in a stylized way in Fig 3.1 (curves for actual calibrated parameter values are shown in Section 4.2). LW enters \mathcal{B} from the left at $(0, q_0)$ and exits from the right at $(1, 0)$. If $k(r + \delta) \geq \eta(y - b - \ell)$, MP enters \mathcal{B} from the top at (q^*, \underline{u}) and exits from the right at $(1, q_1)$. In this case, there exists a nonmonetary equilibrium at $(0, 1)$ and, depending on parameter values, there may also exist monetary equilibria. If $k(r + \delta) < \eta(y - b - \ell)$, MP enters \mathcal{B} from the top at (q^*, \underline{u}) and exits from the bottom at $(0, \bar{u})$. In this case, there exists a nonmonetary

equilibrium at $(0, \bar{u})$, and at least one monetary equilibrium. Generally, steady state equilibrium exists but need not be unique. However, the monetary steady-state may be unique, as was common in calibrated examples.¹³

Conveniently, changes in i shift only the LW curve, while changes in y, η, r, k, δ, b or ℓ shift only the MP curve. In particular, in monetary equilibrium, an increase in i shifts the LW curve toward the origin, decreasing q and increasing u if the equilibrium is unique (or, without uniqueness, in the ‘natural’ low-unemployment equilibrium). The result $\partial q/\partial i < 0$ holds in standard LW models, with fixed α_h , but here there is a general equilibrium multiplier effect: once q falls, u goes down and this reduces α_h , which further reduces q . The result $\partial u/\partial i > 0$ is novel, since the nominal rate has no role in standard MP models and there is no unemployment in standard LW models. This effect captures the idea suggested by Friedman (1977) as discussed in the Introduction. Intuitively, higher i increases the cost of holding money, leading h to economize on real balances; this hurts retail trade, profit, and ultimately employment. Other experiments can be analyzed similarly, and are left as exercises.¹⁴

Proposition 3 *Steady state equilibrium exists. If $k(r + \delta) \geq \eta(y - b - \ell)$, there is a nonmonetary steady state at $(0, 1)$ and there may also exist monetary steady states. If $k(r + \delta) < \eta(y - b - \ell)$, there is a nonmonetary steady state at $(0, \bar{u})$ and at least one monetary steady state. If the monetary steady-state is unique, a rise in i decreases q and increases u , while a rise in y , or a fall in k, b or ℓ , increases q and decreases u .*

¹³Once we have (u, q) , we easily recover $v, \alpha_j, \lambda_j, z$ etc. In particular, given the AD nominal price level $p = M/g(q)$, the budget equation yields x for every h as a function of z and I_e . The employed consume more x , although employed and unemployed consume the same q (but see Liu 2009 for extensions where this is not true). In the case mentioned in fn.7 with many AD goods and utility $x + \Upsilon_e(\mathbf{x})$, standard consumer theory yields individual demand $\mathbf{x} = \mathbf{D}_e(\mathbf{p})$, market demand is $\mathbf{D}(\mathbf{p}) = u\mathbf{D}_0(\mathbf{p}) + (1 - u)\mathbf{D}_1(\mathbf{p})$, and equating this to supply yields a standard system of GE equations that solve for \mathbf{p} . Note that our model does not dichotomize, as some LW models do (see Aruoba et al. 2008), and one cannot generally solve for AD and KW consumption independently.

¹⁴Consider an increase in b , which is the most basic experiment in labor models, like a change in i is the most basic in monetary theory. This shifts the MP curve out, increasing u and reducing q if the equilibrium is unique (or in ‘natural’ equilibrium). The result $\partial u/\partial b > 0$ holds in standard MP models, with fixed R , but now there is a general equilibrium multiplier effect. And $\partial q/\partial b < 0$ is novel, in the sense that standard MP models do not generate a value for money, and standard LW models do not have a role for UI.

4 Quantitative Analysis

We have constructed a framework to analyze labor and goods markets with frictions. The model is tractable, and many results can be established by shifting curves, including the result that increasing inflation raises unemployment through a qualitative channel suggested by Friedman (1977). We now show the theory is amenable to quantitative analysis, by asking how well can it account for the low-frequency behavior of u from 1955-2005, assuming (counterfactually) the only driving force is monetary policy. Although Section 3 considered steady states, here we use the generalization in the Appendix, with a stochastic process for productivity y , and a policy rule that gives next period's nominal rate \hat{i} as a function of i : $\hat{i} = \bar{i} + \rho_i(i - \bar{i}) + \epsilon_i$, $\epsilon_i \sim N(0, \sigma_i)$.

4.1 Parameters and Targets

We choose a model period as one quarter. In terms of parameters, preferences are described by the discount factor β , the value of leisure ℓ , and $v(q) = Aq^{1-\alpha}/(1-\alpha)$. Technology is described by the vacancy cost k , the job-destruction rate δ , and $c(q) = q^\gamma$. Matching is described by $\mathcal{N}(u, v) = Zu^{1-\sigma}v^\sigma$ (truncated to keep probabilities below 1), as in much of the literature following Mortensen-Pissarides (1994), and $\mathcal{M}(B, S) = BS/(B + S)$, following Kiyotaki-Wright (1993). Policy is described by a UI benefit b and a stochastic process for i summarized by $(\bar{i}, \rho_i, \sigma_i)$. Finally, we have bargaining power parameters in MP and KW, η and θ .

We set β so the real interest rate in the model matches the data, measured as the difference between the rate on Aaa bonds and realized inflation. We set $(\bar{i}, \rho_i, \sigma_i)$ to match the average, autocorrelation, and standard deviation of the nominal rate. The parameters k , δ , Z , σ , η and b are fixed using the standard approach in the macro-labor literature (see e.g. Shimer 2005 or Menzio and Shi 2009). Thus, k and δ match the average unemployment rate and UE (unemployment-to-employment) transition rate; Z is normalized so that the vacancy rate is 1; σ is to set match the regression coefficient of v/u on the UE transition rate; η is equated to σ (the Hosios rule); and b is set so UI benefits are half of average wages.

We then set A , α , γ and θ as in the related money literature (see e.g. Aruoba et al. 2008). First, set A and α so the relationship between money demand M/pY and i is the same in the model and data. In the model,

$$\frac{M}{pY} = \frac{M/p}{Y} = \frac{g(q)}{(1-u) \{ \alpha_f [g(q) - c(q)] + y \}}, \quad (21)$$

which depends on i via q and u , and on A and α via the function $g(q)$. Although there are alternative ways to fit this relation, we set A to match average M/pY and α to match the empirical elasticity, using $M1$ as our measure of money.¹⁵ Notice (21) also involves γ in $c(q)$ and θ in $g(q)$. For now we set $\gamma = 1$ (see below), and set θ so the markup in KW matches the data, which according to the retail data summarized by Faig and Jerex (2005) means a target markup of 30 percent (see Aruoba et al. 2008 for more discussion).

The targets discussed above and summarized in Table 1 are sufficient to pin down all but one parameter, the value of leisure ℓ . As is well known, the literature has not reached a consensus on how to set this. For instance, Shimer (2005) assumes $\ell = 0$; Hagedorn and Manovskii (2008) calibrate it using the cost of hiring and find that $(b + \ell)/y = 0.95$; and Hall and Milgrom (2008) calibrate it using consumption data and find that $(b + \ell)/y = 0.71$. Here we follow a different strategy, and set ℓ so that the model implies that, at the business cycle frequency, measured fluctuations in productivity y (holding monetary policy fixed) account for 2/3 of the observed fluctuations in u . While the exact target is somewhat arbitrary, this method reflects a common view, articulated in Mortensen and Nagypal (2006), that productivity is a major but not the only cause of cyclical fluctuations in labor markets.¹⁶

¹⁵We use $M1$ mainly to facilitate comparison with the literature. Although at first blush it may seem $M0$ better suits the theory, one can reformulate this kind of model so that demand deposits circulate in KW, either instead of or along with currency (Berentsen et al. 2007, He et al. 2007, or Chiu and Meh 2009, Li 2009). Also, to pin down the share of the KW market in total output, simply divide nominal KW spending $\mathcal{M}(1, 1 - u)M$ by pY . With our calibration, KW accounts for 42% and AD 58% of GDP (which differs from e.g. Aruoba et al. 2008, since the models are different).

¹⁶We present robustness results on this (and other parameters) below. We also tried some alternative calibration strategies: Berentsen et al. (2008) report results when ℓ is set as in Hagedorn and Manovskii (2008), and when it is set to minimize deviations between predicted and actual u . While the details differ, the overall message is similar.

TABLE 1: CALIBRATION TARGETS

Description	Value
average unemployment u	.006
average vacancies v (normalization)	1
average UE rate λ_h (monthly)	.450
elasticity of λ_h wrt v/u	.280
firm's bargaining power in MP η	.280
average UI replacement rate b/w	.500
average money demand M/pY (annual)	.179
elasticity of M/pY wrt i (negative)	.556
elasticity γ of cost function	1
retail sector markup	.300
average nominal interest rate i (annual)	.074
autocorrelation of i (quarterly)	.989
standard deviation of i	.006
average real interest rate r (annual)	.033

TABLE 2: CALIBRATED PARAMETERS

	Description	Baseline	Markup	Leisure	Elasticity
β	discount factor	.992	.992	.992	.992
ℓ	value of leisure	.504	.517	.514	.491
A	KW utility weight	1.08	1.10	1.07	1.10
α	KW utility elasticity	.179	.211	.179	.105
δ	job destruction rate	.050	.050	.050	.050
k	vacancy posting cost (10^{-4})	8.44	8.68	6.47	8.25
Z	MP matching efficiency	.364	.364	.364	.364
σ	MP matching velasticity	.280	.280	.280	.280
η	MP firm bargaining share	.280	.280	.280	.280
θ	KW firm bargaining share	.275	.225	.275	.275

Table 2 summarizes parameter values. The first column is for the baseline calibration described above. For robustness, we also present three alternative calibrations in the other columns. In the first alternative, labeled Markup, we set θ so that the KW markup is 40% rather than 30%. In the second, labeled Leisure, we set ℓ so that at the business cycle frequency the model accounts for all rather than 2/3 of unemployment volatility in response to fluctuations in y . In the third, labeled Elasticity, we set α so that the elasticity of money demand is -1 rather than -0.556 as in the base case.

Although these alternatives are somewhat arbitrary, they suffice to illustrate how and how much the results depend on parameter values.

4.2 Results

Using the calibrated parameters, we compute equilibrium for the model when i and y follow stochastic processes, as described in the Appendix. Then we input the actual time series for i , holding y constant, and compute the implied path of u . To focus on longer-run behavior, we pass u through an HP filter to eliminate higher-frequency fluctuations. The resulting series is our prediction of what trend unemployment would have been if monetary policy had been the only driving force over the period.

TABLE 3: 1972-1992

	u 1972(1)	u 1982(1)	u 1992(1)	Δ 1972-1982	Δ 1982-1992
Data	5.33	8.16	6.48	2.83	-1.68
Baseline	5.83	7.02	5.96	1.19	-1.06
Markup	5.83	7.97	6.02	2.14	-1.95
Leisure	5.83	7.91	6.01	2.08	-1.90
Elasticity	5.83	7.55	6.02	1.72	-1.53
All data is passed through a 1600 HP-filter					

For the baseline calibration, Fig. 4.1 plots time-series for the actual and counterfactual trend u (as well as the unfiltered series). It is apparent that money could have been responsible for a large part of the movements in u : the model indicates e.g. that changes in i alone can account for over 40% of the 2.83 increase in u between 1972 and 1982, and over 60% of the 1.68 point decline between 1982 and 1992, as seen in Table 3. Between 1992 and 2005, i also accounts for the overall decline in u , if not all the fluctuations. The 1960s are the only big subperiod where actual and counterfactual u move in opposite directions.¹⁷ Fig. 4.2 shows the scatter plot of

¹⁷There is clearly no hope of explaining movements in u in the 60s as a function of movements in i alone, since theory predicts $\partial u / \partial i < 0$. We can however say the decline in u in this subperiod was due to other factors, like increased productivity, and slack monetary policy actually prevented unemployment from falling by more. Quantitatively, we need to increase y from 1 to 1.0275 during the 60s to explain lower u despite higher i during that decade. We leave more detailed exploration of this idea to future work.

actual u and i in blue, and of counterfactual predicted u and i in red, where we note that the relationship between counterfactual u and i is close to the linear regression on the actual data. Fig. 4.3 replaces the nominal rate with inflation, and delivers similar results.

Table 3 also summarizes the results from the alternative calibrations. As one can see, monetary policy accounts for a larger fraction of the movements in trend u if we target a higher retail mark-up, if we assume that y shocks account for a larger fraction of fluctuations in u at the business cycle frequency, or if we assume money demand is more elastic. Fig. 4.4 shows the case where ℓ is higher. To understand the results, note that y and i have different effects on R , but conditional on having a given effect on R , they have the same effect on u . If u responds more to y , as it does when ℓ is higher, then it also responds more to i . If the elasticity of money demand is higher or the mark-up is higher, a change in i shock has a larger effect on R . One can also interpret these in terms of shifts and slopes of the MP and LW curves (which conveys the economic intuition, even though Section 3.3 only considered steady states). If ℓ increases, e.g., the MP curve flattens out, so that a shift in LW from a change in i induces a larger increase in u , as shown in Fig. 4.5.

From these counterfactual analyses, we conclude that monetary policy can be important for understanding the low-frequency performance of the US labor market, at least over the period 1955-2005. This conclusion is independent of nominal rigidities, imperfect information, and any other channel that may or may not be relevant at high frequencies. Moreover, we conclude that the importance of monetary policy on the performance of the labor market in the long run is stronger, the higher is the mark-up in the retail sector, the higher is the elasticity of money demand, and the higher is the contribution of productivity shocks to cyclical unemployment.

4.3 Financial Innovation

The baseline model generates a relationship between nominal interest rates and money demand that closely resembles its empirical counterpart up until the 1990s. However,

during the 1990s, M/pY is systematically lower for all nominal interest rates – the money demand curve shifts down – and for this period the baseline parameters do not match the data well. This is a concern, because as we argued in the previous subsection, the shape of money demand plays an important role in determining the effect of i on u . Therefore, here we carry out the same analysis of the effect of monetary policy on trend unemployment in a simple generalization of the model that is better able to reproduce the empirical money demand data.¹⁸

Recall the model discussed in Section 2, where ω is the probability a meeting in KW is anonymous, and hence the probability money is needed for trade. We now allow ω to differ before and after 1990. This is meant to capture in a crude but reasonable way the idea that the downward shift in money demand curve was due to innovations in payments, such as the proliferation of credit cards in retail, and perhaps other innovations, like ATMs, sweep accounts etc. that allow households to economize on liquid balances. We keep $\omega = 1$ from 1955-1990, and set $\omega = 0.62$ after 1990 to match average M/pY in the latter period, with the other parameters set to match the same targets as the baseline calibration.¹⁹ Given these parameter values, we compute equilibrium under the assumption that a one-time unexpected change in ω occurred, which is again crude but we think illustrative. Then we feed in the actual path for i and compute the predicted path for u .

Fig. 4.6 presents the money demand curve generated by the model with financial innovation in purple and the baseline model in red, as well as actual money demand in green. The model with financial innovation generates a money demand curve that has a higher mean and elasticity before 1990, and a much lower mean after 1990. Generally, with financial innovation, money demand in the model is closer to the

¹⁸This exercise is very similar in spirit to the analysis by Guerrieri and Lorenzoni (2008) in a related model.

¹⁹By comparison, Aruoba et al. (2008) argue for $\omega = 0.88$, to match Klee's (2008) finding that shoppers use credit cards (as opposed to cash, checks and debit cards) for 12% of supermarket transactions in the scanner data. This is close to the 16% Cooley and Hansen (1991) report was found in earlier consumer survey data). While future work on matching micro payments data is clearly desirable, we think calibrating to aggregate money demand suffices for the points we want to make here.

data. Figure 4.7 shows actual trend unemployment u in blue, and the path for trend u predicted by the model with financial innovation (in purple by the baseline model in red). Clearly, the model with financial innovation implies monetary policy accounts for a much larger fraction of the movement in u during the earlier part of the sample, and about the same fraction in the latter part.

Overall, once we allow for financial innovation, the model fits much better the empirical money demand curve, and implies that monetary policy accounts for a larger fraction of the movement in trend unemployment over the period. This result provides another a robustness check on the earlier results. Additionally, this extension implies that the observed shift in money demand is likely to reduce the impact of monetary policy on the labor market in the future. In Figure 4.8 the black line shows the path for trend u assuming $\omega = 0.62$ over the entire period 1955-2005. In this case, the inflation of the 1970s would have had a much smaller effect on u , e.g., and hence we can predict that in the future, assuming money demand does not shift back, inflation will not lead to as large an increase in unemployment as we observed during stagflation, say.

5 Comparison with Cash in Advance

A question often comes up in this kind of research is, why do we need monetary theory with microfoundations? At one level, we obviously do not *need* the search-and-bargaining approach to study the effect of money on unemployment, since some of the papers mentioned in the Introduction use cash-in-advance (henceforth CIA) models. One doesn't even need a model in the modern sense – we could use the IS-LM approach combined with Okun's Law. The interesting issue is not one of *need*, but whether it makes a difference for the results when one uses, say, the search-and-bargaining or CIA framework. To discuss this issue, here we consider the model with an otherwise frictionless and competitive goods market – i.e. no search or bargaining

– but impose a CIA constraint.²⁰

We will compare the models in two ways: examine the mechanisms through which money matters analytically; and use calibrated versions to contrast results numerically. For the first approach, without going through the rudimentary details, the setup with CIA but otherwise no frictions in KW generates a demand for q given by

$$v'(q) = (1 + i)c' \left(\frac{q}{1 - u} \right). \quad (22)$$

The LHS is the MRS between q and x , and the RHS is the opportunity cost of q in terms of x , including the interest cost $1 + i$ and the marginal cost c' evaluated at the equilibrium quantity produced by a firm active in the market (i.e. one that is matched with a worker). An increase in i raises the cost due to the CIA constraint, while an increase in u raises marginal cost for each active f , since there are fewer of them, increasing the price of q . Hence an increase in either i or u reduces demand for KW goods.

By comparison, in our search and bargaining model, demand for the KW good satisfies

$$v'(q) = \left(1 + \frac{i}{\alpha_h} \right) g'(q). \quad (23)$$

There are two differences between (22) and (23). First, because of search frictions, h only gets to trade in the KW market with probability α_h , making the effective interest rate i/α_h , instead of i . Second, f has market power in KW, making the effective price $g'(q)$ rather than $c'(q)$, where $g(q)$ is the bargaining solution described above. In our model, an increase in i reduces the demand for q , as in the CIA model, but the effect is larger, given $\alpha_h < 1$ and given $g(q)$ is typically less convex than $c(q)$. Moreover, in our model an increase in u affects q by lowering the probability of trade, which is different from the CIA model, where an increase in u simply means that each active f has to produce more to get the same total q .

²⁰There is no model in the literature that does exactly this, although Andofatto et al. (2003) and Cooley and Quadrini (2004) do use CIA models. However, to give the reduced-form approach a chance, we really need to have both cash and credit goods, since a simple CIA model cannot match well the empirical money demand.

Additionally, in both models the entry (vacancy posting) decision of f is based on expected revenue R , but in the CIA model,

$$R = c' \left(\frac{q}{1-u} \right) \frac{q}{1-u} - c \left(\frac{q}{1-u} \right) + y. \quad (24)$$

The first term on the RHS is revenue from KW sales, the second is the cost, and the last term is revenue from AD sales. An increase in the demand for q increases R in the CIA model by increasing the difference between the revenue and cost associated with the KW good, and an increase in u increases R by increasing marginal cost and hence equilibrium price of the KW good.

By comparison, in our model

$$R = \alpha_f [g(q) - c(q)] + y. \quad (25)$$

An increase in q here increases R by raising the surplus f gets from KW sales, $g(q) - c(q)$. This is similar to the effect of q on R in the CIA model, except with bargaining the magnitude of the effect depends not only on the shape of the cost function but depends also on the utility function and bargaining power. Additionally, an increase in u raises R in our model by increasing the firm's probability of KW trade α_f , an effect that is completely absent in the CIA model. We conclude that the channels via which q affects u , as well as the channels via which u affects q , are qualitatively different in the models, as is the impact of a change in i .

We now turn to a quantitative comparison. First, consider the case in which cost $c(q) = q^\gamma$ is linear: $\gamma = 1$. In the CIA model, an increase in i increases the opportunity cost of holding money, which reduces the demand for KW goods, but with linear cost and a competitive market, the price of q goods and hence R are completely unaffected by this decline in demand. Therefore, in the CIA model, with linear cost, an increase in i has no effect on the incentive for f to open vacancies in the MP market and hence no effect on u . By contrast, in our model f has market power and price exceeds cost in KW. Thus, R falls with a decline in demand in our model, so an increase in i reduces vacancies and employment. In our calibrated model, increasing inflation

from 0 to 10% raises u from 5.2 to 7.4 across steady states, but does literally nothing to u in the CIA model.

That was for linear cost. Suppose now cost is convex: $\gamma > 1$. Then the price of KW goods exceeds average cost, and R does depend on demand even in the CIA model. Thus a fall in demand for q reduces vacancies and employment even in the CIA model. In a calibrated version (see Table 4), increasing inflation from 0 to 10% raises u from 5.4 to 6.6 when we set $\gamma = 1.05$, and from 5.4 to 6.8 when we set $\gamma = 1.10$. By comparison, the same policy increases u from 5.2 to 7.9 when we set $\gamma = 1.05$, and from 5.1 to 8.7 when we set $\gamma = 1.10$. Thus our model generates much bigger effects, mainly because the share of the surplus accruing to f in KW is determined differently, and is both larger and more sensitive to changes in demand.

TABLE 4: CALIBRATED PARAMETERS

	$\gamma = 1$		$\gamma = 1.05$		$\gamma = 1.1$	
	BMW	CIA	BMW	CIA	BMW	CIA
β	.992	.992	.992	.992	.992	.992
ℓ	.504	.480	.511	.497	.517	.513
A	1.08	1.01	1.13	1.04	1.21	1.08
α	.179	.030	.156	.001	1.13	.001
δ	.050	.050	.050	.050	.050	.050
k	8.44	4.11	8.56	4.25	8.68	4.40
Z	.364	.364	.364	.364	.364	.364
σ	.720	.720	.720	.720	.720	.720
η	.280	.280	.280	.280	.280	.280
θ	.275	—	.250	—	.275	—

In both models, increasing the convexity of $c(q)$ magnifies the response of u to changes in i , but also dampens the response of M/PY to changes in i , and makes it harder to match the empirical elasticity of money demand. Intuitively, the higher is γ , the smaller the effect of an increase in i on q and hence on M/PY . Quantitatively, the CIA model can match the elasticity of M/PY for $\gamma = 1$, but fails for $\gamma = 1.05$ or higher: for $\gamma \geq 1.05$, there are no parameters of the utility function $v(q)$ that make the CIA model look like the empirical money demand curve. In contrast, our model can match the empirical money demand curve for $\gamma = 1, 1.05$ or 1.10 . This is

because, in our model, h faces an effective interest rate of i/α_h , rather than simply i , and hence an increase in i has a larger effect on q and M/PY . So to the extent that one is disciplined by money demand, and not free to pick cost functions totally arbitrarily, our model predicts a bigger quantitative impact of monetary policy on the labor market than the CIA model. Fig 5.1 summarizes the results.

These comparisons show that the search-and-bargaining foundations for money matter a lot. First, because frictions give market power to f in KW, i has a stronger effect on u in our setup for any specification of $c(q)$. Second, because frictions imply that h trades only probabilistically in KW, i has a stronger effect on M/pY in our model, and hence we are better able to match the empirical money demand elasticity.²¹ For the CIA model to match the elasticity of money demand, $c(q)$ must be close to linear, and hence the Phillips curve must be approximately flat. Our model can match the elasticity for more general cost functions, and for any of them, including the linear case, we can get large effects of i on u . We conclude that while one may not *need* microfoundations, they do *matter* for the results.

6 Conclusion

This paper has studied the relation between unemployment and monetary variables like inflation or nominal interest rates. We first documented that these variables are positively related in the low frequency data. We then developed a very tractable framework where money and unemployment are both modeled using microfoundations based on search-and-bargaining, providing a unified theory for analyzing labor and goods markets. We then showed the model is amenable to quantitative analysis by asking how well it can account for unemployment behavior when the only impulse is monetary policy. We found that changes in monetary policy alone can generate a sizable fraction of historical movements in unemployment, although probably will generate less in the future given shifts in money demand explicable in terms of innovations in payments. Finally, we asked if it matters, qualitatively and quantitatively,

²¹See also Telyukova and Visshers (2009).

whether one uses monetary economics based on search-and-bargaining microfoundations or monetary economics based on an ad hoc cash-in-advance specification. The answer is yes – it does matter.

Appendix: The Dynamic-Stochastic Model

At the beginning of a period, the state is $s = (u, i, y)$, where u is unemployment, i the nominal rate and y productivity. The state s was known in the previous AD market, including the return on nominal bonds maturing this period. Although these bonds are not traded in equilibrium, i matters because it pins down the expected return on real balances $\hat{\rho}(s) = \mathbb{E}[\rho(\hat{s})|s]$ via the no-arbitrage condition $1 = \beta(1 + i)\hat{\rho}(s)$. The nominal rate and productivity follow exogenous (independent) processes:

$$\begin{aligned}\hat{i} &= \bar{i} + \rho_i(i - \bar{i}) + \epsilon_i, \epsilon_i \sim N(0, \sigma_i) \\ \hat{y} &= \bar{y} + \rho_y(y - \bar{y}) + \epsilon_y, \epsilon_y \sim N(0, \sigma_y)\end{aligned}$$

Unemployment behaves as follows. In MP, each unemployed h finds a job with probability $\lambda_h[\tau(s)]$ and f with a vacancy fills it with probability $\lambda_f[\tau(s)]$, where $\tau(s) = v/u$ and $v = v(s)$ was set in the previous AD market. Therefore, at the beginning of KW,

$$\hat{u}(s) = u - u\lambda_h[\tau(s)] + (1 - u)\delta.$$

When h and f meet in MP, $w(s)$ is determined by generalized Nash bargaining, but is paid (in units of x) in AD; $w(s)$ can be renegotiated in MP each period.

In KW market, h meets f with probability $\alpha_h[Q(s)]$ and f meets h with probability $\alpha_f[Q(s)]$, where $Q(s) = 1/[1 - \hat{u}(s)]$, whence $q(z, s)$ and $d(z, s)$ are determined according to generalized Nash bargaining, where z denotes real balances held by h . After KW, in the AD market, the realization of \hat{s} becomes known, f liquidates inventories, pays wages and dividends, and create $v(\hat{s})$ vacancies for the next MP. Also, h chooses $z(\hat{s})$, and government collects $T(\hat{s})$, pays b , and announces \hat{i} .

In MP, taking as given the equilibrium wage function $w(s)$, the value functions for h are

$$\begin{aligned} U_0^h(z; s) &= V_0^h(z; s) + \lambda_h[\tau(s)] \{V_1^h[z, w(s); s] - V_0^h(z; s)\} \\ U_1^h(z; s) &= V_1^h[z, w(s); s] - \delta \{V_1^h[z, w(s); s] - V_0^h(z; s)\}. \end{aligned}$$

In KW, taking as given the equilibrium terms of trade $q(z; s)$ and $d(z; s)$,

$$\begin{aligned} V_0^h(z; s) &= \alpha_h \left[\frac{1}{1 - \hat{u}(s)} \right] \{v[q(z; s)] - \hat{\rho}(s)d(z; s)\} + \hat{\rho}(s)[z - d(z; s)] + \mathbb{E}W_0^h(0; \hat{s}) \\ V_1^h(z, w; s) &= \alpha_h \left[\frac{1}{1 - \hat{u}(s)} \right] \{v[q(z; s)] - \hat{\rho}(s)d(z; s)\} + \hat{\rho}(s)[z - d(z; s)] + \mathbb{E}W_1^h(0, w; \hat{s}), \end{aligned}$$

using the linearity of $W_e^h(\cdot; \hat{s})$. Finally, in AD,

$$\begin{aligned} W_0^h(z; \hat{s}) &= z + b + \ell + \Delta(\hat{s}) - T(\hat{s}) + \max_{\hat{z} \geq 0} \{-\hat{z} + \beta U_0^h(\hat{z}; \hat{s})\} \\ W_1^h(z, w; \hat{s}) &= z + w + \Delta(\hat{s}) - T(\hat{s}) + \max_{\hat{z} \geq 0} \{-\hat{z} + \beta U_1^h(\hat{z}; \hat{s})\}. \end{aligned}$$

Let $z(\hat{s})$ be solution to the above maximization, $d(s) = d[z(s); s]$ and $q(s) = q[z(s); s]$.

For f , in MP, taking as given $w(s)$, the value functions are

$$\begin{aligned} U_0^f(s) &= \lambda_f[\tau(s)] V_1^f[w(s); s] \\ U_1^f(s) &= (1 - \delta) V_1^f[w(s); s]. \end{aligned}$$

In KW, taking as given $q(z; s)$, $d(z; s)$ and $z(s)$,

$$V_1^f(w; s) = \alpha_f \left[\frac{1}{1 - \hat{u}(s)} \right] \{\hat{\rho}(s)d(s) - c[q(s)]\} + \beta \mathbb{E}W_1^f(0, y, w; \hat{s}).$$

And in AD,

$$\begin{aligned} W_0^f(\hat{s}) &= \max\{0, -k + U_0^f(\hat{s})\} \\ W_1^f(z, y, w; \hat{s}) &= y + z - w + \beta U_1^f(\hat{s}). \end{aligned}$$

In MP the surplus of a match is

$$S(s) = V_1^h[z, w; s] + V_1^f[w; s] - V_0^h(z; s),$$

where we note that both z and w vanish on the right hand side. The bargaining solution implies $w(s)$ is such that

$$\begin{aligned} V_1^h[z, w(s); s] - V_0^h(z; s) &= (1 - \eta)S(s) \\ V_1^f[w(s); s] &= \eta S(s). \end{aligned}$$

In KW, bargaining solution implies that $d(z; s) = z$ and $q(z; s)$ is such that $\hat{\rho}(s)z = g[q(z; s)]$, with $g(q)$ as defined in the text.

The transition probability function $\mathcal{P}(\hat{s}; s)$ is constructed from the laws of motion for i , y , and u in the obvious way. Then a Recursive Equilibrium is a list of functions $S(s)$, $q(s)$, $\tau(s)$, and $\mathcal{P}(\hat{s}; s)$ such that:

$$\begin{aligned} S(s) &= y + b - \ell + \alpha_f \left[\frac{1}{1-\hat{u}(s)} \right] \{g[q(s)] - c[q(s)]\} + \beta \mathbb{E}\{1 - \delta - (1 - \eta)\lambda_h[\tau(\hat{s})]\} S(\hat{s}); \\ \frac{i}{\alpha_h \left[\frac{1}{1-\hat{u}(s)} \right]} &= \frac{v'[q(s)]}{g'[q(s)]} - 1; \\ k &= \beta \lambda_f[\tau(s)] \eta S(s); \end{aligned}$$

and \mathcal{P} is consistent with the law of motion for (i, u, y) . Now standard methods in quantitative macroeconomics allow us to solve for the equilibrium functions numerically. See <http://www.wwz.unibas.ch/witheo/aleks/BMWII/BMWII.html> for details, including programs for calibration and simulation.

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